abstract
This thesis presents a control-theoretic analysis of several types of mean-field models proposed in the literature for modeling and control of large-scale multi-agent systems, including robotic swarms. These mean-field models are forward equations of stochastic processes, and their analysis is motivated by the fact that as the number of agents tends to infinity, the empirical measure associated with the agents converges to the solution of these models. Hence, the problem of transporting a swarm of agents from one configuration to another can be posed as a control problem for the forward equation of the process which determines the time evolution of the swarm probability density. This thesis presents such an analysis for three types of mean-field models. First, this thesis considers the case in which the agents' states evolve on a finite state space according to a continuous-time Markov chain (CTMC), and the forward equation is an ordinary differential equation (ODE). The finite-time controllability, asymptotic controllability, and stabilization of the forward equation, with the agents' transition rates defined as the control parameters is investigated. Second, the controllability and stabilization problem for systems of advection-diffusion-reaction partial differential equations (PDEs) is studied. In this case, the agents' states evolve according to a hybrid-switching diffusion process. The controllability of this model to sufficiently regular target probability densities is constructively proved, with the agents' velocity field and reaction rates defined as the control parameters. Spatially-dependent coefficients to asymptotically stabilize these target distributions are also constructed for this model. Third, mean-field feedback laws, with reaction rates as control parameters, are constructed in order to stabilize a swarm of non-holonomic agents to a probability density with a disconnected support. Global Asymptotic stability of the resulting semilinear PDE model is established. Fourth, this thesis considers a controllability and optimal control problem in the more general case where the agent dynamics are given by a nonlinear discrete-time control system. Beyond these theoretical results, this thesis also considers numerical optimal transport for control-affine systems. It is shown that finite-volume approximations of the associated PDEs lead to well-posed transport problems on graphs as long as the control system is controllable everywhere.