Mechanical Engineering Doctoral Defense

Set-Valued Methods for Reachability Analysis and Estimation of Nonlinear Dynamical Systems

School for Engineering of Matter, Transport and Energy

Mohammad Khajenejad

Advisor: Sze Zheng Yong

Abstract

The goal of this thesis research is to contribute to the design of set-valued methods, i.e., algorithms that leverage a set-theoretic framework that can provide a powerful means for control designs for very general classes of uncertain nonlinear dynamical systems, and in particular, to develop set-valued algorithms for constrained reachability problems and estimation. We propose novel fixed-order set-valued (i.e., hyperball and interval valued) observers for different classes of nonlinear systems, including (1) parameter varying, (2) Lipschitz continuous and (3) Decremental Quadratic Constrained nonlinearities, with unknown inputs and bounded noise signals that simultaneously find bounded sets of states and unknown inputs that contain the true states and unknown inputs and are compatible/consistent with the measurement outputs. In addition, we provide (necessary) and sufficient conditions for the existence, stability and optimality of the designed observers. Moreover, we design state and unknown input estimation as well as mode detection for (4) hidden mode switched linear systems with bounded-norm noise and unknown inputs. To address this, we propose a multiple-model approach to obtain a bank of mode-matched set-valued observers in combination with a novel mode observer based on elimination. Furthermore, we address the problem of designing interval observers for (5) partially unknown nonlinear systems with bounded noise signals. Leveraging affine abstraction methods and non-linear decomposition functions, as well as a data-driven function over-approximation/abstraction approach to over-estimate the unknown dynamic model, our proposed observer recursively computes the maximal and minimal elements of the interval estimates that are proven to frame the true augmented states. Then, using observed output/measurement signals, the observer iteratively shrinks the intervals. Moreover, the observer updates the over-approximation model of the unknown dynamics. Finally, we propose a tractable family of remainder-from decomposition functions, that their existence is proven to be sufficient conditions for mixed-monotonicity of a broad-range of not necessarily smooth, constrained and unconstrained, continuous and discrete-time bounded-error dynamical systems. Moreover, we develop a set-inversion algorithm that along with the proposed decomposition functions have several applications, e.g., in approximation of the reachable sets for bounded-error, constrained, continuous and/or discrete-time systems, as well as in guaranteed state estimation.