## Mechanical Engineering Doctoral Defense

Set-Valued Methods for Learning, Control and Estimation

School for Engineering of Matter, Transport and Energy

Zeyuan Jin

Advisor: Sze Zheng Yong

## **Abstract**

In this thesis, set-membership methods are designed for learning unknown system dynamics, feedback control and state estimation problems. First, the thesis developed approaches for finding upper and lower bounds of the vector fields of complex system dynamics to simplify the models for control and estimation tasks.

Specifically, optimization-based approaches are proposed for finding piecewise-affine over-approximations of the nonlinear models with uncertain coefficients, including with polytopic partitions/subregions to reduce their conservativeness. Given only prior noisy sampled data when precise mathematical models are unavailable, two data-driven set-membership learning approaches are proposed under different assumptions over continuity of the system, namely under assumptions of Lipschitz continuity and differentiability with bounded Jacobian matrices. Since both methods fall under the umbrella of non-parametric learning approaches which often lack scalability, down-sampling techniques are proposed to reduce the computation complexity of the algorithm. Once the set-membership models are obtained, it was shown that any model (passive) invalidation guarantees for the over-approximated system also hold for the original system.

Second, the problem of state and unknown terrain estimation is addressed, where unknown terrain parameters, e.g., terrain stiffness, are inferred from motion through vehicle-terrain interaction. In particular, a state and model interval observer is designed for terrain estimation based on set-membership estimation, where the goal is to find set-valued estimates (in the form of hyperrectangles or intervals) of the states and unknown terrain parameters.

Finally, robust data-driven control barrier functions (CBF-DDs) are proposed to guarantee robust safety of unknown continuous control systems despite worst-case realizations of generalization errors. The aforementioned non-parametric data-driven approaches are leveraged to learn guaranteed upper and lower bounds of the unknown time-derivative of control barrier function from the data set to formulate/obtain a safe input set for a given state. By incorporating the safe input set into an optimization-based controller, system safety can be ensured for all times.