Mechanical Engineering Dissertation Defense

Analysis, Estimation, and Control of Partial Differential Equations Using Partial Integral Equation Representation

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Abstract

When solving analysis, estimation, and control problems for Partial Differential Equations (PDEs) via computational methods, one must overcome the following hurdles: (a) there is no universal parametric representation of such systems; (b) the parameters are often unbounded differential operators; and (c), auxiliary constraints such as Boundary conditions and continuity conditions must be enforced via ad-hoc steps.

To overcome these hurdles, an alternative representation of PDEs called the `Partial Integral Equation' (PIE) representation is proposed. Primarily, the PIE representation addresses the non-universal parametrization of PDEs by using bounded linear operators, called Partial Integral (PI) operators, as parameters, overcoming the two former hurdles. Furthermore, for admissible PDEs, the PIE representation is unique and does not require any auxiliary constraints --- resolving the final hurdle.

Using this PIE representation, mathematical and computational tools based on convex-optimization and Lyapunov methods can be developed to solve Control problems such as internal stability, Input-to-Output L_2-gain, H_{∞}-optimal state observer, and H_{∞}-optimal full state-feedback controller design for linear PDEs in one spatial dimension.

The PIE representation for a PDE is obtained by finding a unique unitary map from the states of the PIE to the states of the PDE. This map shows a PDE and its associated PIE have equivalent system properties, including wellposedness, internal stability, and I/O behavior. Furthermore, this unique map also allows us to construct a well-defined dual representation that can be used to solve optimal control problems for a PDE.

Once the PIE associated with a PDE is obtained, Lyapunov functions (or storage functions) are parametrized by positive PI operators to obtain a solvable convex formulation of the above-stated Control problems. Lastly, the methods proposed here are applied to various PDE systems to demonstrate the application.

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