# Mechanical Engineering Dissertation Defense

## "Value and Policy Approximation for Two-player General-sum Differential Games"

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# Abstract

Human-robot interactions can often be formulated as general-sum differential games where the equilibrial policies are governed by Hamilton-Jacobi-Isaacs (HJI) equations. Solving HJI PDEs faces the curse of dimensionality (CoD). While physics-informed neural networks (PINNs) alleviate CoD in solving PDEs with smooth solutions, they fall short in learning discontinuous solutions due to their sampling nature. This causes PINNs to have poor safety performance when they are applied to approximate values that are discontinuous due to state constraints. This dissertation aims to improve the safety performance of PINN-based value and policy models.

The first contribution of the dissertation is to develop learning methods to approximate discontinuous values. Specifically, three solutions are developed: (1) hybrid learning uses both supervisory and PDE losses, (2) value-hardening solves HJIs with increasing Lipschitz constant on the constraint violation penalty, and (3) the epigraphical technique lifts the value to a higher-dimensional state space where it becomes continuous. Evaluations through 5D and 9D vehicle and 13D drone simulations reveal that the hybrid method outperforms others in terms of generalization and safety performance.

The second contribution is a learning-theoretical analysis of PINN for value and policy approximation. Specifically, by extending the neural tangent kernel (NTK) framework, the thesis explores why the choice of activation function significantly affects the PINN generalization performance, and why the inclusion of supervisory co-state data improves the safety performance.

The last contribution is a series of extensions of the hybrid PINN method to address real-time parameter estimation problems in incomplete-information games. Specifically, a Pontryagin-mode PINN is developed to avoid costly computation for supervisory data. The key idea is the introduction of a costate loss, which is cheap to compute yet effectively enables the learning of important value changes and policies in space-time. Building upon this, a Pontryagin-mode neural operator is developed to achieve state-of-the-art (SOTA) safety performance across a set of differential games with parametric state constraints. We demonstrate the utility of the resultant neural operator in estimating player constraint parameters during incomplete-information games.

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