Mechanical Engineering Dissertation Defense

Extension of the Partial Integral Equation Representation for Analysis of Multivariate and Nonlinear PDEs

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Abstract

Physical phenomena evolving in time and space are commonly modeled by partial differential equations (PDEs). For a broad class of linear one-dimensional PDEs, these equations admit an equivalent Partial Integral Equation (PIE) representation, parameterized by Partial Integral (PI) operators. In this representation, analysis and control problems can be formulated as convex optimization problems over PI operators and solved numerically, yielding systematic computational methods for stability analysis and controller synthesis.

Despite its success for linear one-dimensional PDEs, the PIE framework does not readily accommodate more general classes of systems. Its construction relies on inversion of the highest-order differential operator under the prescribed boundary conditions, preventing direct application to problems with, for example, periodic boundary conditions. Furthermore, extending the framework to multidimensional and nonlinear PDEs requires fundamentally new operator parameterizations and system representations.

Several extensions of the PIE framework are proposed to overcome these limitations. For one-dimensional PDEs with general boundary conditions, including periodic boundaries, the PIE state is augmented with components spanning the null space of the differential operator, restoring an invertible mapping between PDE and PIE states. For PDEs on multidimensional domains, a general class of boundary conditions is parameterized using multivariate PI operators, and inverse differential operators are constructed within this framework, yielding a multivariate PIE representation that naturally extends existing analysis and control techniques. Finally, multivariate PI operators are used to parameterize polynomial functionals on infinite-dimensional states, leading to a nonlinear PIE framework capable of representing PDEs with polynomial nonlinearities. Based on this representation, a convex optimization approach is developed to certify exponential stability.

These extensions substantially broaden the scope of the PIE framework, enabling systematic stability analysis of PDEs with general boundary conditions, multiple spatial variables, and polynomial nonlinearities. Their effectiveness is demonstrated through several examples.

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